REPORT DOCUMENTATION PAGE

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate only, other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (07804-0188), Washington, DC 20503.

1. AGENCY USE ONLY *(LEAVE BLANK)*	2. REPORT DATE 4 June 1999	3. REPORT TYPE AND DATES COVERED Professional Paper

4. TITLE AND SUBTITLE Effects of Control Allocation Algorithms on a Nonlinear Adaptive Design	5. FUNDING NUMBERS
6. AUTHOR(S) Anthony B. Page Marc L. Steinberg	

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Air Warfare Center Aircraft Division 22347 Cedar Point Road, Unit #6 Patuxent River, Maryland 20670-1161	8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) Naval Air Systems Command 47123 Buse Road, Unit IPT Patuxent River, Maryland 20670-1547	10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution is unlimited.	12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

The effect of the choice of control allocation algorithm used in conjunction with a nonlinear adaptive control law is examined. In particular, an existing backstepping control law design is modified to allow the incorporation of various control allocation algorithms. Simulation results are presented for two separate adaptive control law designs and for a nonadaptive two-loop dynamic inversion controller. With the control law designs fixed, tracking maneuvers are simulated using four separate control allocation routines - Direct Allocation, Discrete Time Direct Allocation, Pseudo-inverse, and Weighted Pseudo-Inverse. System performance is then examined as a function of control allocation method. As expected, even though the remainder of the control law design remains fixed, system performance is directly impacted by the choice of control allocation algorithm. Further, some of the important control law/control allocation interactions are identified and their effects on overall system performance analyzed.

14. SUBJECT TERMS Control Allocation Algorithms Nonlinear Adaptive Design	15. NUMBER OF PAGES 11
	16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION OF ABSTRACT UL

AIAA-99-4282

EFFECTS OF CONTROL ALLOCATION ALGORITHMS
ON A NONLINEAR ADAPTIVE DESIGN

Anthony B. Page[*] and Marc L. Steinberg[†]
Naval Air Systems Command, Patuxent River, MD 20670

Abstract

The effect of the choice of control allocation algorithm used in conjunction with a nonlinear adaptive control law is examined. In particular, an existing backstepping control law design is modified to allow the incorporation of various control allocation algorithms. Simulation results are presented for two separate adaptive control law designs and for a non-adaptive two-loop dynamic inversion controller. With the control law designs fixed, tracking maneuvers are simulated using four separate control allocation routines – Direct Allocation, Discrete Time Direct Allocation, Pseudo-Inverse, and Weighted Pseudo-Inverse. System performance is then examined as a function of control allocation method. As expected, even though the remainder of the control law design remains fixed, system performance is directly impacted by the choice of control allocation algorithm. Further, some of the important control law/control allocation interactions are identified and their effects on overall system performance analyzed.

Introduction

Basic control law algorithms and control allocation approaches are frequently treated as separate disciplines. Papers on control laws often use fairly simple control allocation approaches or assume the same number of controls as controlled variables. Papers on control allocation often look primarily at the ability to attain commanded moments. Yet, as both flight control laws and control allocation techniques become increasingly complex, the possibilities of adverse or unexpected interactions increase. While there have been some efforts that have looked at the advanced control law/control allocation problem (e.g., References 1-5), many questions still exist. This is particularly true with regard to potential interactions with adaptive or other highly nonlinear control laws.

Further, for some advanced control algorithms, there is the possibility of improvements through greater integration between the control law and control allocation algorthms. For example, in the case of an adaptive control law, should the control allocation parameters remain fixed or should the control allocation be updated in some way to deal with errors in control effectiveness caused by model uncertainty or aircraft damage? For an adaptive controller that explicitly identifies control effectiveness, there may be a reasonably straightforward way of performing the update (although there are a number of practical difficulties involved such as effector collinearities). For a direct adaptive controller, the situation is much less clear. Some direct adaptive approaches have no known way of extracting control effectiveness changes. Some other approaches, like backstepping adaptive control, have pseudo-estimates of control parameters, but the parameter estimation is done to meet a total system stability criterion rather than to optimize the error in estimation. As a result, the estimates are not guaranteed to converge to the actual values over time and it is not clear whether the use of such values in control allocation would be beneficial. This is further complicated by the fact that different inputs, different flight conditions, or different levels of model uncertainty may have substantial effects on the values of the estimates and the resulting control law interactions.

This paper examines the interactions of two adaptive backstepping control law designs and a non-adaptive two-loop Dynamic Inversion control law with four different control allocation approaches. Dynamic Inversion is an approach based on canceling the aircraft's natural dynamics and substituting desired dynamic behavior[2]. Backstepping[6] is a Lyapunov approach that can be used to design stable adaptive control laws for a wide range of systems. The basic concept behind backstepping is to use some states as virtual controls to control other states. The primary benefits of this type of controller are that it allows a wide array of nonlinearities to be incorporated in the

controller design, and it has proven nominal stability and convergence of error. Previous work in the application of adaptive backstepping designs to the aircraft control problem[7,8] have assumed the same number of control as controlled variables. Concurrent to the work presented herein, Reference 9 examines the use of backstepping control with redundant control effectors. While Reference 9 uses only a Weighted Pseudo-Inverse solution to the control allocation problem, this paper endeavors to begin an examination of the dynamic interactions between various control allocation schemes and nonlinear adaptive controllers of the type examined in References 7-9.

If there are no limits on the control effectors, then the particular control allocation algorithm employed is usually considered to be of little consequence. That is to say, any legitimate control allocation algorithm will be able to realize the desired control effect. However, if an estimate of the control effectiveness is used, different control allocation algorithms could indeed produce different responses even when no constraints are placed on the control effectors. Furthermore, when control effector rate and/or position limits are considered, the control allocation algorithm becomes extremely important in determining system response even with perfect knowledge of the control effectiveness parameters.

There have been many recent advances in control allocation (e.g., References 3, 5, 10, and 11). For the initial examination of the effect of control allocation on system behavior under the control of a nonlinear adaptive control law, four control allocation techniques will be implemented: Direct Allocation[10], Discrete Time Direct Allocation[11], Pseudo- Inverse, and Weighted Pseudo-Inverse. While a more exhaustive array of control allocation algorithms could be implemented, those chosen should prove sufficient to demonstrate some of the important control law/control allocation interactions and the profound impact of these interactions on system performance.

Aircraft Model

The chosen nonlinear aircraft dynamics model (Eq. 1) is similar to that used in References 7-9. Here, $\Delta\alpha = \alpha - \alpha_0$ and the control positions are measured from their trim values. The seven independent control effectors are taken to be the left and right horizontal tails, left and right ailerons, collective leading edge flaps, collective trailing edge flaps, and collective rudders. Some of the key simplifications made in this model are constant velocity, no lift and drag effects of the control surfaces,

$$
\begin{pmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \\ \dot{\alpha} \\ \dot{\beta} \\ \dot{\phi} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} l_\beta\beta + l_q q + l_r r + (l_{\beta a}\beta + l_{ra}r)\Delta\alpha + l_p p - i_1 qr \\ m_a\Delta\alpha + m_q q + i_2 pr - m_a p\beta + m_a(g_0/V)(\cos\theta\cos\phi - \cos\theta_0) \\ n_\beta\beta + n_r r + n_p p + n_{pa}p\Delta\alpha - i_3 pq + n_q q \\ q - p\beta + z_a\Delta\alpha + (g_0/V)(\cos\theta\cos\phi - \cos\theta_0) \\ y_\beta\beta + p(\sin\alpha_0 + \Delta\alpha) - r\cos\alpha_0 + (g_0/V)\cos\theta\sin\phi \\ p + q\tan\theta\sin\phi + r\tan\theta\cos\phi \\ q\cos\phi - r\sin\theta \end{pmatrix}
$$

$$
+ \begin{pmatrix} l_{\delta_{ht}} & l_{\delta_{hr}} & l_{\delta_{ail}} & l_{\delta_{air}} & 0 & 0 & l_{\delta_{rud}} \\ m_{\delta_{ht}} & m_{\delta_{hr}} & m_{\delta_{ail}} & m_{\delta_{air}} & m_{\delta_{lef}} & m_{\delta_{tef}} & m_{\delta_{rud}} \\ n_{\delta_{ht}} & n_{\delta_{hr}} & n_{\delta_{ail}} & n_{\delta_{air}} & 0 & 0 & n_{\delta_{rud}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \delta_{htl} \\ \delta_{htr} \\ \delta_{ail} \\ \delta_{air} \\ \delta_{lef} \\ \delta_{tef} \\ \delta_{rud} \end{pmatrix} \quad (1)
$$

and no higher frequency dynamics (e.g., actuators). The control surface movements are however restricted by the rate and position limits given in Table 1. The nominal model parameters are given in Table 2 for trimmed flight at 30,000 feet Mach 0.7. Table 3 gives the parameter values for trimmed flight at 40,000 feet Mach 0.6. Trim values for the horizontal tails, leading edge flaps, and trailing edge flaps are –0.68°, 4.41°, and 5.56° for Table 1, and –0.97°, -0.70°, and 10.73° for Table 2 – those not specified are zero. These parameters are somewhat, but not completely, unlike those of an F-18 trimmed at the corresponding flight conditions.

Table 1: Effector Rate and Position Limits

Effector	Position Limits (deg)	Rate Limits (deg)
Horizontal Tails	+10.5, -24	±40
Ailerons	+45, -25	±100
Leading Edge Flaps	+33, -3	±15
Trailing Edge Flaps	+45, -8	±18
Rudder	+30, -30	±82

Control Law Designs

For comparative purposes, two adaptive control law designs and one non-adaptive design will be implemented. The first adaptive design includes estimation of the full set of 17 control effectiveness parameters. The second adaptive design entails defining a set of three pseudo controls and then estimating only five pseudo control effectiveness parameters.

Full Control Estimation

This design essentially follows that of Reference 8 and is similar to that of References 7 and 9. However, the

Table 2: Aircraft Model Parameters for trimmed flight at 30,000 ft., Mach 0.7 (all angles and angular rates are assumed to be in radians and radians/sec respectively)

$l_\beta = -11.04$	$l_q = 0$	$l_r = 0.4164$	$l_{\beta\alpha} = -19.72$	$l_{r\alpha} = 4.709$	$l_p = -1.4096$	$z_\alpha = -0.6257$	$y_\beta = -0.1244$
$m_\alpha = -5.432$	$m_{\dot\alpha} = -0.1258$	$m_q = -0.3373$	$n_\beta = 2.558$	$n_r = -0.1122$	$n_p = -0.0328$	$n_{p\alpha} = -0.0026$	$n_q = 0$
$l_{\delta_{ht}} = 6.3176$	$l_{\delta_{hr}} = -6.3176$	$l_{\delta_{ail}} = 7.9354$	$l_{\delta_{air}} = -7.9354$	$l_{\delta_{rud}} = 1.8930$	$i_1 = 0.7966$	$i_2 = 0.9595$	$i_3 = 0.6914$
$m_{\delta_{ht}} = -4.5176$	$m_{\delta_{hr}} = -4.5176$	$m_{\delta_{ail}} = -0.8368$	$m_{\delta_{air}} = -0.8368$	$m_{\delta_{hf}} = -1.2320$	$m_{\delta_{hf}} = 0.9893$	$m_{\delta_{rud}} = 0$	$g_0 = 32\,ft/sec^2$
$n_{\delta_{ht}} = 0.2814$	$n_{\delta_{hr}} = -0.2814$	$n_{\delta_{ail}} = -0.0698$	$n_{\delta_{air}} = -0.0698$	$n_{\delta_{rud}} = -1.7422$	$V = 696\,ft/sec$	$\alpha_0 = 0.0681$	$\theta_0 = 0.0681$

Table 3: Aircraft Model Parameters for trimmed flight at 40,000 ft., Mach 0.6 (all angles and angular rates are assumed to be in radians and radians/sec respectively)

$l_\beta = -7.0104$	$l_q = 0$	$l_r = 0.3529$	$l_{\beta\alpha} = -16.4015$	$l_{r\alpha} = 1.0461$	$l_p = -0.7331$	$z_\alpha = -0.2876$	$y_\beta = -0.0700$
$m_\alpha = -1.4592$	$m_{\dot\alpha} = -0.0177$	$m_q = -0.1286$	$n_\beta = 1.3612$	$n_r = -0.0619$	$n_p = -0.0177$	$n_{p\alpha} = 0.0696$	$n_q = 0$
$l_{\delta_{ht}} = 2.7203$	$l_{\delta_{hr}} = -2.7203$	$l_{\delta_{ail}} = 4.2438$	$l_{\delta_{air}} = -4.2438$	$l_{\delta_{rud}} = 0.8920$	$i_1 = 0.7966$	$i_2 = 0.9595$	$i_3 = 0.6914$
$m_{\delta_{ht}} = -1.9782$	$m_{\delta_{hr}} = -1.9782$	$m_{\delta_{ail}} = -0.3183$	$m_{\delta_{air}} = -0.3183$	$m_{\delta_{hf}} = -0.4048$	$m_{\delta_{hf}} = 0.3034$	$m_{\delta_{rud}} = 0$	$g_0 = 32\,ft/sec^2$
$n_{\delta_{ht}} = 0.1262$	$n_{\delta_{hr}} = -0.1262$	$n_{\delta_{ail}} = -0.0963$	$n_{\delta_{air}} = 0.0963$	$n_{\delta_{rud}} = -0.8018$	$V = 581\,ft/sec$	$\alpha_0 = 0.1447$	$\theta_0 = 0.1447$

following design does not contain the fuzzy logic mitigation of actuator saturation as was done in Reference 8. The aircraft equations of motion (Eq. 1) can be put in the following form:

$$\dot{y} = \Phi_0(x_1) + \Phi(x_1)w_1 + B(x_1)\omega$$
$$\dot{\omega} = \psi_0(x) + \psi_1(x)w_2 + D(x, w_u)u \quad (2)$$
$$\dot{\eta} = q_0(x) + q_1(x)w + q_2(x, w_u)u$$

In this form, y is a vector of the outputs that will be controlled, ω is the virtual control vector used to control y, η is the vector of uncontrolled states, w_1, w_2, and w_u are the vectors of unknown parameters, x_1 is a subset of the state vector x, and u is the vector of control effector commands where,

$$y = (\phi, \alpha, \beta)^T$$
$$\omega = (p, q, r)^T \quad (3)$$
$$\eta = \theta$$
$$x_1 = (\alpha, \beta, \phi, \theta)$$
$$w_1 = (z_\alpha, y_\beta)^T$$
$$w_2 = (l_\beta, l_p, l_q, l_r, l_{\beta\alpha}, l_{r\alpha}, m_\alpha, m_{\dot\alpha}, m_q, n_\beta, n_r, n_p, n_{p\alpha}, n_q)^T$$
$$w_u = (l_{\delta_{ht}}, l_{\delta_{hr}}, l_{\delta_{ail}}, l_{\delta_{air}}, l_{\delta_{rud}}, m_{\delta_{ht}}, m_{\delta_{hr}}, m_{\delta_{ail}}, m_{\delta_{air}}, \ldots$$
$$m_{\delta_{hf}}, m_{\delta_{hf}}, m_{\delta_{rud}}, n_{\delta_{ht}}, n_{\delta_{hr}}, n_{\delta_{ail}}, n_{\delta_{air}}, n_{\delta_{rud}})^T$$

We will next define the error, e, and a function s that combines error and integrated error,

$$e = y - y_c$$
$$s = e + K_0 x_s \quad (4)$$
$$\dot{x}_s = e$$

where K_0 is a positive definite symmetric matrix and y_c is the output of a command generator that is a linear, stable, 3rd order system. It will be useful to define the derivative of S as:

$$\dot{s} = \Phi(x_1)w_1 + B(x_1)(\omega_d + \widetilde{\omega}) + v$$
$$\omega = \omega_d + \widetilde{\omega} \quad (5)$$
$$v = \Phi_0 - \dot{y}_c + K_0 e$$

In Equation 5, ω_d represents the desired value of the virtual controls. However, since these are states and not directly controllable effectors, there will be an error $\widetilde{\omega}$.

We will next choose ω_d through the use of the following Lyapunov function:

$$U_1 = (s^T s + \widetilde{w}_1^T L_1 \widetilde{w}_1)/2$$
$$\widetilde{w}_i = w_i - \hat{w}_i \quad (6)$$

where L_1 is a positive definite diagonal matrix and $\hat{w}_i$ is the estimate of w_i.

Taking the derivative of U_1 yields:

$$\dot{U}_1 = s^T[\Phi(x_1)w_1 + B(x_1)(\widetilde{\omega} + \omega_d) + v(x_1, \dot{y}_c)] + \widetilde{w}_1^T L_1 \widetilde{w}_1 \quad (7)$$

ω_d can then be chosen as:

$$\omega_d = B^{-1}(x_1)[-K_{11}s - \Phi^T(x_1)\hat{w}_1 - v(x_1, \dot{y}_c)] \quad (8)$$

where K_{11} is a positive definite diagonal matrix.

As a result of that choice of ω_d,

$$\dot{U}_1 = -s^T K_{11} s + \widetilde{w}_1^T \left[\Phi^T s + L_1 \dot{\widetilde{w}}_1 \right] + \widetilde{\omega}^T B^T(x_1) s \qquad (9)$$

If we put $\dot{\omega}_d$ in the following form:

$$\dot{\omega}_d = \psi_{od} - \psi_{1d} w_1 + \psi_{2d} w_2 \qquad (10)$$

then,

$$\dot{\widetilde{\omega}} = \psi_{0a} + \psi_{1a} \hat{w}_1 + \psi_{2a} \widetilde{w}_2 + D(x, w_u) u$$
$$\psi_{0a} = \psi_0 - \psi_{0d} + \psi_{1d} \hat{w}_1 + \psi_{2a} \hat{w}_2$$
$$\psi_{1a} = \psi_{1d} \qquad (11)$$
$$\psi_{2a} = \psi_1 - \psi_{2d}$$

We will next chose a 2nd Lyapunov function such that,

$$U_2 = U_1 + (\widetilde{\omega}^T \widetilde{\omega} + \widetilde{w}_2^T L_2 \widetilde{w}_2 + \widetilde{w}_u^T L_3 \widetilde{w}_u)/2 \qquad (12)$$

where L_1 and L_2 are positive definite diagonal matrices. Taking the derivative of U_2 gives:

$$\dot{U}_2 = -s^T K_{11} s + \widetilde{w}_1^T \left[\Phi^T s + L_1 \dot{\widetilde{w}}_1 + \psi_{1a}^T \widetilde{\omega} \right] + \widetilde{w}_2 \left[L_2 \dot{\widetilde{w}}_2 + \psi_{2a}^T \widetilde{\omega} \right]$$
$$+ \widetilde{w}_u^T L_3 \dot{\widetilde{w}}_u + \widetilde{\omega}^T \left[B^T s + \psi_{0a} + Du \right] \qquad (13)$$

We will next chose the following control law:

$$\hat{D} u = D(x, \hat{w}_u) u = (-B^T s - \psi_{0a} - K_{22} \widetilde{\omega}) = m_{des} \qquad (14)$$

and the following parameter update laws:

$$\dot{\hat{w}}_1 = L_1^{-1} (\Phi^T s + \psi_{1a}^T \widetilde{\omega})$$
$$\dot{\hat{w}}_2 = L_2^{-1} \psi_{2a}^T \widetilde{\omega} \qquad (15)$$
$$\dot{\hat{w}}_u = L_3^{-1} \Psi_u^T \widetilde{\omega}$$

where Ψ_u is chosen such that,

$$[D(x, w_u) - D(x, \hat{w}_u)] u = \Psi_u(x, u) \widetilde{w}_u \qquad (16)$$

As a result, since K_{11} and K_{22} are positive definite diagonal matrices,

$$\dot{U}_2 = -s^T K_{11} s - \widetilde{\omega}^T K_{22} \widetilde{\omega} \le 0 \qquad (17)$$

Following the approach of Reference 7, it can be shown that for the nominal system, s, $\widetilde{\omega}$, and e all tend to zero as $t \to \infty$.

Furthermore, it is desirable to bound the parameter estimates on some interval to help avoid singularities in the control law. To this end, a projection algorithm is employed similar to that described in Reference 9. Suppose that each component $\hat{w}_{ik}$ of $\hat{w}_i$ ($i = 1, 2, u$) is known to lie in an open interval $\overset{\circ}{W}_{ik} = (l_{ik}, m_{ik})$. Then a modified adaptation law is chosen as ($i = 1, 2, u$),

$$\dot{\hat{w}}_{ik} = \begin{cases} \mu_{ik} & \text{if } \hat{w}_{ik} \in \overset{\circ}{W}_{ik} \text{ or } \hat{w}_{ik} = m_{ik} \text{ and } \mu_{ik} \le 0 \\ & \quad\quad \text{ or } \hat{w}_{ik} = l_{ik} \text{ and } \mu_{ik} \ge 0 \\ 0 & \text{otherwise} \end{cases} \qquad (18)$$

where $\mu_{ik} = \dot{\hat{w}}_{ik}$ from equation 15.

Pseudo Control Estimation

A separate adaptive control law can be obtained if we define the following pseudo controls (u_p) and corresponding control effectiveness (D_p):

$$D_p u_p = \begin{pmatrix} l_{\delta_{lat}} & 0 & l_{\delta_{dir}} \\ 0 & m_{\delta_{lon}} & 0 \\ n_{\delta_{lat}} & 0 & n_{\delta_{dir}} \end{pmatrix} \begin{pmatrix} \delta_{lat} \\ \delta_{lon} \\ \delta_{dir} \end{pmatrix} \qquad (19)$$

with,

$$w_{u_p} = (l_{\delta_{lat}}, l_{\delta_{dir}}, m_{\delta_{lon}}, n_{\delta_{lat}}, n_{\delta_{dir}})^T \qquad (20)$$

The control law design then proceeds exactly as described for Full Control Estimation except that D_p, u_p, and w_{up} should replace D, u, and w_u in equations 2 through 18. In order to determine the actual control commands from the pseudo control commands, another relationship is required. Namely,

$$\hat{D} u = u_p = \hat{D}_p^{-1} m_{des} \qquad (21)$$

where $\hat{D}$ now indicates an estimate of the full control effectiveness matrix that is either given a priori or generated on-line from a source other than the nonlinear adaptive controller.

Non-Adaptive

A non-adaptive controller that is essentially a two-loop dynamic inversion controller is obtained by simply turning off the parameter updates ($\dot{\hat{w}}_i = 0$ ($i = 1, 2, u$)) for either of the preceding adaptive designs.

4

Control Allocation Techniques

In the above control law derivations, the control vector (u) is chosen such that either,

$$\hat{D}_{3\times 7} u_{7\times 1} = m_{des\,3\times 1} \qquad (22)$$

or,

$$\hat{D}_{3\times 7} u_{7\times 1} = u_{p\,3\times 1} = \hat{D}_{p\,3\times 3}^{-1} m_{des\,3\times 1} \qquad (23)$$

where the subscripts (m x n) represent the matrix and vector dimensions for the chosen design of Equations 1 and 2. For Full Control Estimation (Equation 22), the elements of $\hat{D}$ are updated by the parameter update law of Equation 15. For Pseudo Control Estimation (Equation 23), it is the elements of $\hat{D}_p$ that are updated by the parameter update law of Equation 15. When there are no constraints on the control vector, Equations 22 and 23 will generally have an infinite number of solutions. However, when control constraints are considered, these equations can either have an infinite number of solutions, no solution, or a unique solution. The solution of Equation 22 or 23 for the control vector (u) is the control allocation problem. Only a brief description of each control allocation algorithm will be presented. For more detailed information, consult the references cited.

Direct Allocation

Durham[10] recognized that existing control allocation schemes did not fully exploit the capabilities of the redundant control effectors they were commanding. Durham therefore defined the Attainable Moment Subset (AMS) to be the set of all moments capable of being produced by a given set of control effectors constrained to move only within certain bounds. In Direct Allocation (DA), the control effector commands are calculated as a scaled version of the deflections that yield the maximum attainable moment in the desired direction. That is, given the direction of any m_{des}, the controls are found that lie on the boundary of the AMS in that direction. If the desired moment is smaller than maximum attainable moment in the given direction, the control vector is simply scaled down (preserving the direction) such that Equation 22 or 23 is satisfied. If the desired moment is greater than or equal to the maximum attainable moment, then the control vector is set equal to that on the boundary of the AMS. Direct allocation insures that all moments attainable by the control effectors are indeed realizable by the control allocation algorithm. For those moments outside the AMS, the direction of the moment is preserved.

Discrete Time Direct Allocation

Direct Allocation only considers position limiting of the control effectors. A straightforward discrete time extension of Direct Allocation allows the incorporation of effector rate limiting to be considered as well. In Discrete Time Direct Allocation[11, 12] (also referred to as Moment-Rate Allocation) the problem essentially can be defined as the solution to,

$$\hat{D}\,\Delta u = \Delta m_{des} \qquad (24)$$

Now, the limits on the control, Δu, are taken as the more restrictive of 1) the global position limits or 2) the effector rate limits multiplied by Δt. That is, the control effectors are limited by how far they can travel during the next time increment (Δt). Therefore, the ΔAMS can be defined as the set of all moments producible over the next time step. With this problem formulation, the same algorithms used in Direct Allocation can be applied to Equation 24.

Discrete Time Direct Allocation (DTDA) is prone to a form of controller windup[11, 12]. To alleviate this problem various restoring techniques have been offered which have the ability to drive the control allocation solution so as to optimize a given objective function. The results presented herein use the minimum norm restoring technique as described in References 11 and 12. This technique drives the solution to that which minimizes the 2-norm of the control vector.

Pseudo-Inverse and Weighted Pseudo-Inverse

A simple and computationally efficient solution to the control allocation problem is the use of a Weighted Pseudo-Inverse (WPI). In particular, the solution of Equation 22 or 23 that minimizes the following quadratic function:

$$J = u^T W u \qquad (25)$$

where W is an appropriate weighting matrix, is given by[13]:

$$u = W^{-1}(\hat{D})^T [\hat{D} W^{-1}(\hat{D})^T]^{-1} m_{des} \qquad (26)$$

While Equation 26 provides a unique solution to Equation 22 or 23, it does not consider any limitations on the control effectors. This solution technique is used as the sole control allocator in a similarly designed adaptive controller in Reference 9. When W=I, the solution is termed the Pseudo-Inverse (PI) solution.

Simulation Results

The aircraft simulation model is defined in Equation 1. The control effector rate and position limits are given in Table 1. The aircraft model parameters for two separate flight conditions are given in Tables 2 and 3. The three control laws are defined as specified previously with the controller gains as follows:

$$K_0=0.4I, K_{11}=I, K_{22}=4I, L_1=0.2I, L_2=0.2I, L_3=0.2I \quad (27)$$

where I is the identity matrix of the proper dimension.

The initial condition estimates of the stability and control parameters for the Full Control Estimation design were chosen to vary ±50% from their nominal values as indicated in Table 4. For the Pseudo Control Estimation design, the initial pseudo control effectiveness matrix ($\hat{D}_p$) is chosen as the identity

Table 4: Initial Conditions for Parameter Estimates

IC Number	$\hat{w}_i$ $(i=1,2)$	$\hat{w}_u$
1	$0.50w_i$	$0.50w_u$
2	$1.00w_i$	$0.50w_u$
3	$1.50w_i$	$0.50w_u$
4	$0.50w_i$	$1.00w_u$
5	$1.00w_i$	$1.00w_u$
6	$1.50w_i$	$1.00w_u$
7	$0.50w_i$	$1.50w_u$
8	$1.00w_i$	$1.50w_u$
9	$1.50w_i$	$1.50w_u$

matrix and the estimated control effectiveness matrix ($\hat{D}$) is fixed at the value determined by the control effectiveness parameter estimates given in Table 4. The parameter estimates, $\hat{w}_{ik}$ (i = 1, 2, u), are bound by the projection algorithm of Equation 18 to be within 75% of their nominal values with the exception of $\hat{l}_{\delta_{dr}}$ and $\hat{n}_{\delta_{lat}}$ which are bound between –0.2 and 0.2.

Both medium and large amplitude single and multi-axis doublet maneuvers were simulated at both flight conditions starting with the initial estimates given Table 4. The piecewise constant inputs to the command generator were of the form: $y^*(t) = (\phi_1^*, \alpha_0+\alpha_1^*, \beta_1^*)$ for $0 \le t < 4$ seconds, $y^*(t) = (\phi_2^*, \alpha_0+\alpha_2^*, \beta_2^*)$ for $4 \le t < 8$ seconds, and $y^*(t) = (0, \alpha_0, 0)$ for $8 \le t < 15$ seconds. Table 5 lists the command generator inputs for the six

maneuvers considered. Unless otherwise noted, the initial conditions chosen for the command generator are $y_c(0) = \dot{y}_c(0) = \ddot{y}_c(0) = 0$, except $\alpha_c(0)=\alpha_0$, and the command generator poles are placed at –1.5 and –1.5±1.06i.

Table 5: Command Generator Inputs

Maneuver Number	ϕ_1^* (deg)	ϕ_2^* (deg)	α_1^* (deg)	α_2^* (deg)
1	45	-45	0	0
2	90	-90	0	0
3	0	0	20	-10
4	0	0	40	-20
5	30	-30	15	-7.5
6	60	-60	30	-15

The only difference among the simulations for each control allocation method is the algorithm used to solve Equation 22 or 23 for the control vector. The algorithms used for both Direct Allocation and Discrete Time Direct Allocation were adapted from Reference 14. Weighted Pseudo-Inverse allocation is given explicitly by Equation 26 where the weighting matrix is chosen as, W= diag(30, 30, 3, 3, 8, 9, 4). Simulation results are also presented for the Pseudo Inverse (PI) solution (i.e.,W=I).

Simulations of all six maneuvers were carried out at both flight conditions for all nine initial conditions, three control law designs, and with each of the four control allocation techniques. All simulations were run for 15 seconds. Since the maneuvers of Table 5 should be completed within 12 seconds, it is of interest to note how many individual simulation runs ended with large tracking errors (average norm of the tracking error greater than 10 degrees) between 13 and 15 seconds. The existence of large tracking errors during the last two seconds is used as an indication the controller was unable to perform adequate tracking for the specified maneuver.

Table 6 shows the number of simulations terminating with large tracking errors for each control allocation technique. If the data of Table 6 is broken down further, it is found that the runs ending with large tracking error occur for two distinct situations. For the non-adaptive controller, the entries in Table 6 all occur at the 30,000 ft., Mach 0.7 flight condition. On the other hand, the entries in Table 6 for both adaptive designs all occur at the 40,000 ft., Mach 0.6 flight condition.

Of the cases indicated Table 6 for the non-adaptive design, the vast majority occur for initial conditions where the control effectiveness is underestimated.

Table 6: Number of Cases Terminating with Large Tracking Errors

	DA	PI	WPI	DTDA
Non-Adaptive	5	5	6	5
Full Control Estimation	2	1	0	0
Pseudo Control Estimation	9	5	0	9

Therefore, it is likely that the control law/control allocator combination over commands the actuators leading to poor damping and large tracking errors. The data in Table 6 reveals that only three cases yield large terminal tracking errors for the Full Control Estimation design (2 for Direct Allocation and 1 for Pseudo-Inverse allocation). The Pseudo Control Estimation design, however, has significant trouble at the lower dynamic pressure flight condition. A notable exception is the Weighted Pseudo-Inverse allocation method that had no cases ending with large tracking errors for either of the adaptive designs.

In order to present meaningful tracking error statistics, two reduced data sets will be used. The first data set results from removing the nine distinct cases that lead at least one allocation scheme to terminate with large tracking errors for either the non-adaptive design or the Full Control Estimation design. The word "case" in this context refers to a particular combination of flight condition, maneuver number, and initial condition number. The results for the average 2-norm of the tracking error over all maneuvers in this data set are given in Table 7. The data in Table 7 shows that all allocation techniques perform reasonably well for the non-adaptive design with pseudo-inverse allocation winning by a narrow margin. However, when Full Control Estimation adaptation is employed, Weighted Pseudo-Inverse allocation vaults into the lead followed closely by Discrete Time Direct Allocation. The values for the Pseudo Control Estimation design are presented in Table 7 to illustrate the average norm of the tracking error is reduced by 30% in going from the Full Control Estimation to the Pseudo Control Estimation control law design for Weighted Pseudo-Inverse allocation. This represents a reduction of over 60% from the smallest average tracking error norm for the non-adaptive design. The Pseudo Control Estimation results

given in Table 7 for the remaining control allocation techniques contain data from maneuvers ending with large tracking errors and are hence not directly comparable.

The second data set results from removing the eleven additional cases that also lead at least one allocation scheme to end with large tracking errors for the Pseudo Control Estimation design. The average norms of the tracking errors for this data set are given in Table 8. The data in Table 8 illustrates the same trends as the data in Table 7 with the added information that, when it works, Pseudo Control Estimation outperforms Full Control Estimation for each of the control allocation technique tested.

On the surface, these results are quite surprising since the given Weighted Pseudo-Inverse allocation technique produces admissible solutions for only about 10% by volume of the AMS. An admissible solution is defined as one for which the estimated attained control effect ($\hat{D}u_{lim}$) equals the desired control effect (m_{des} or u_p). Here, u_{lim} represents the controls as constrained by their position limits. Perhaps more importantly, weighted pseudo-inverse allocation also produces admissible solutions for only 10% by volume of the ΔAMS as defined by the control effector rate limits. By contrast (and by definition) Discrete Time Control Allocation produces admissible solutions for 100% of the AMS as well as for 100% of the ΔAMS.

A deeper understanding of the contributing control law/control allocation interactions can be gained by detailed examination of a single maneuver. For this examination, a 360° rolling maneuver at constant angle-of-attack and zero sideslip will be commanded at the lower dynamic pressure flight condition of Table 3. The inputs to the command generator are $y^*(t) = (90°t, \alpha_0, 0)$ for $0 \le t < 4$ seconds and $y^*(t) = (360°, \alpha_0, 0)$ for $4 \le t < 15$ seconds. The command generator poles for this maneuver are placed at –4.5 and –4.5±3.18i.

Figures 1a-1h show the simulation results for the Pseudo Control Estimation control law with WPI control allocation where the initial parameter estimates

Table 7: Average Error Norm for Reduced Maneuver Set (Nine Cases Removed)

	DA	PI	WPI	DTDA
Non-Adaptive	2.6145	2.3670	2.5135	2.5118
Full Control Estimation	1.7801	1.7269	1.2003	1.3888
Pseudo Control Estimation	2.0436*	1.8861*	0.8373	2.5450*

*Includes cases with large tracking errors at termination

Table 8: Average Error Norm for Reduced Maneuver Set (Twenty Cases Removed)

	DA	PI	WPI	DTDA
Non-Adaptive	2.6145	2.3670	2.5135	2.5118
Full Control Estimation	1.4043	1.4567	1.1689	1.2404
Pseudo Control Estimation	0.7439	1.0214	0.7257	0.8352

American Institute of Aeronautics and Astronautics

are taken to correspond to IC Number 1 from Table 4. In Figures 1a and 1b, the simulated roll angle and angle-of-attack (solid lines) are seen to track the commanded values (dotted lines) acceptably well. The tracking errors are explicitly plotted in Figure 1c. Skipping to Figures 1e and 1f, the control surface deflections exhibit little rate or position saturation with the exception of the rudder. The rudder is either rate or position limited during most of the maneuver.

The changes in the parameter estimates during the maneuver are shown in Figure 1g. Though these changes are mostly smooth, the effects of the parameter bounding algorithm can be seen to kick in during periods of rudder position saturation. Figure 1d shows the pseudo control command and Figure 1h gives the difference between the desired pseudo control and the estimated control effect produced by the control allocator. The nonzero values of Figure 1h indicate that the desired pseudo control is outside the ΔAMS for the control allocator.

Figures 2a-2h show simulation results for the same situation as Figure 1, except that DTDA is used as the control allocation technique. Figures 2a – 2c show the tracking performance to be unacceptable. Figures 2e and 2f show severe limiting of control surfaces. The behavior of the control law during periods of control surface position limiting is one of the key interactions responsible for the relatively poor results for DTDA. Consider the system behavior between 2 and 4 seconds. During this time, five control surfaces reach their position limits indicating the desired control effect lies either outside the AMS or on the boundary of the AMS. With the controls this severely limited, the system does adequately respond to changes in the desired control effect and hence the parameter estimates all head to their limiting values. It is exactly this type of behavior that lead to the use of fuzzy logic to reduce the adaptation and controller gains when the controls become limited[8]. As the parameter estimates become

saturated, the controller looses damping and the response is greatly degraded.

To understand the other contributing interaction, it is necessary to examine the difference between the pseudo control commands and the estimated control effect. Table 9 shows this information for the first 0.04 seconds of the maneuver for both WPI and DTDA control allocation. For the first 0.02 seconds, both allocation techniques produce admissible controls. At 0.03 seconds, the DTDA allocator still produces admissible controls but the WPI allocator does not. Finally, neither allocator produces admissible solutions at 0.04 seconds.

The important interaction lies with the how the control allocation routines deal with unattainable values of the pseudo control commands. While, DTDA preserves the direction of the desired effect by design, WPI can produce unexpected results. At 0.04 seconds, the DTDA routine provides 87% of the desired control effect while the WPI solution provides 104% of the δ_{lat} desired and only 73% of the desired δ_{dir}. The WPI solution sacrifices directional control power and actually yields more lateral control than desired. Since the WPI solution generates artificial lead in the lateral axis, the magnitudes of the pseudo control commands are actually reduced while performing the given rolling maneuver and the stability of the control law is enhanced.

However, the lead produced by the WPI solution could serve to destabilize the control law for another maneuver. For example, if the command generator inputs are taken to be $y^*(t) = (35°\sin(2t), \alpha_0, 0)$ for $0 \leq t < 10$, then the roll angle responses shown in Figure 3 result. This time, the DTDA technique exhibits adequate tracking while the WPI solution generates large errors.

Table 9: Difference Between Pseudo Control commands and Estimated Control Effect

	Desired Control Effect (u_p)			Desired Control Effect – Estimated Control Effect for WPI (u_p-$\hat{D}u_{lim}$)			Desired Control Effect – Estimated Control Effect for DTDA (u_p-$\hat{D}u_{lim}$)		
Time (sec)	δ_{lat}	δ_{lon}	δ_{dir}	$\Delta\delta_{lat}$	$\Delta\delta_{lon}$	$\Delta\delta_{dir}$	$\Delta\delta_{lat}$	$\Delta\delta_{lon}$	$\Delta\delta_{dir}$
0.01	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.02	0.0210	0.0000	0.0031	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.03	0.0614	0.0000	0.0089	-0.0011	0.0000	0.0010	0.0000	0.0000	0.0000
0.04	0.1173	0.0000	0.0171	-0.0051	0.0000	0.0046	0.0150	0.0000	0.0022

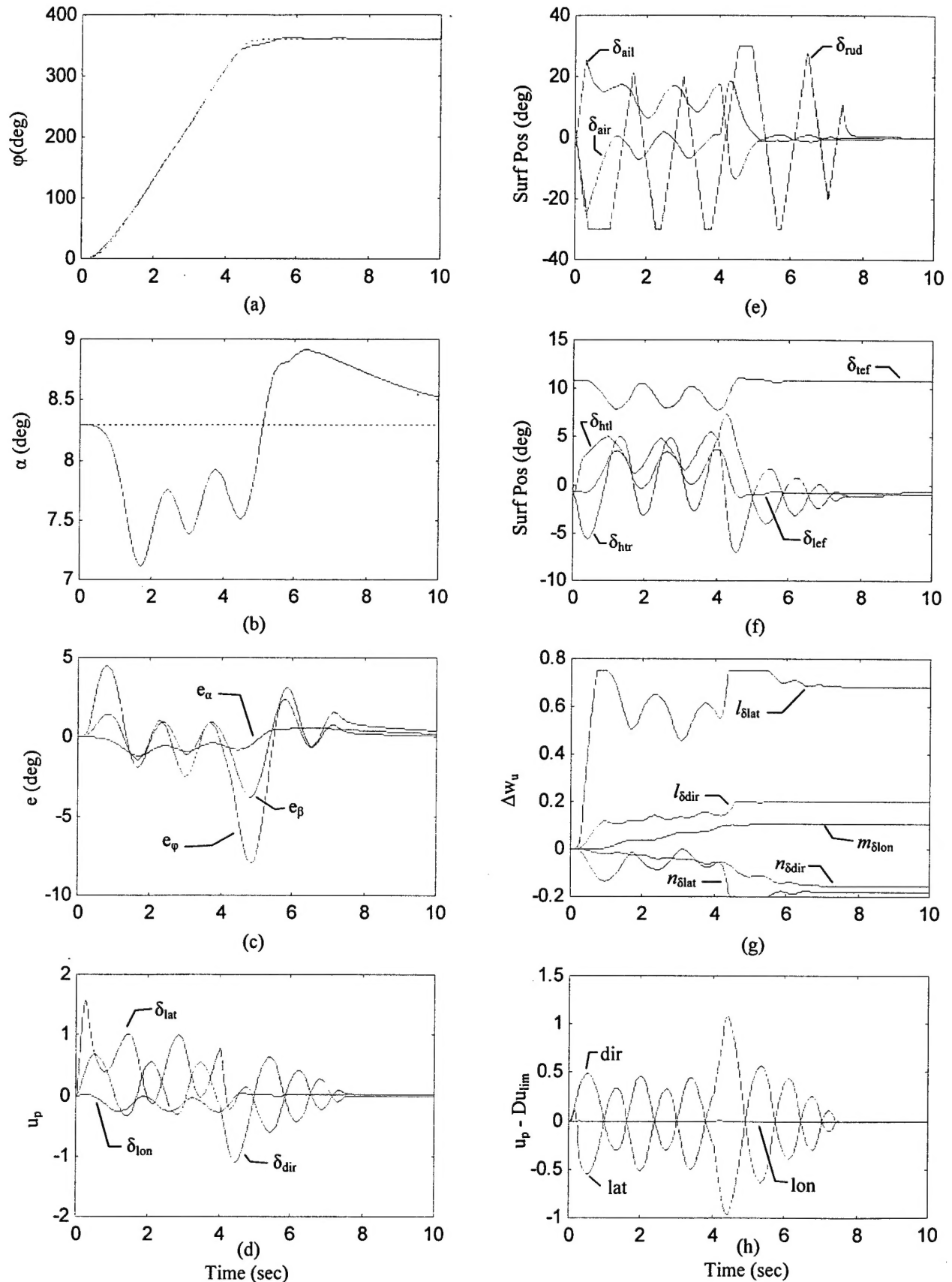American Institute of Aeronautics and Astronautics

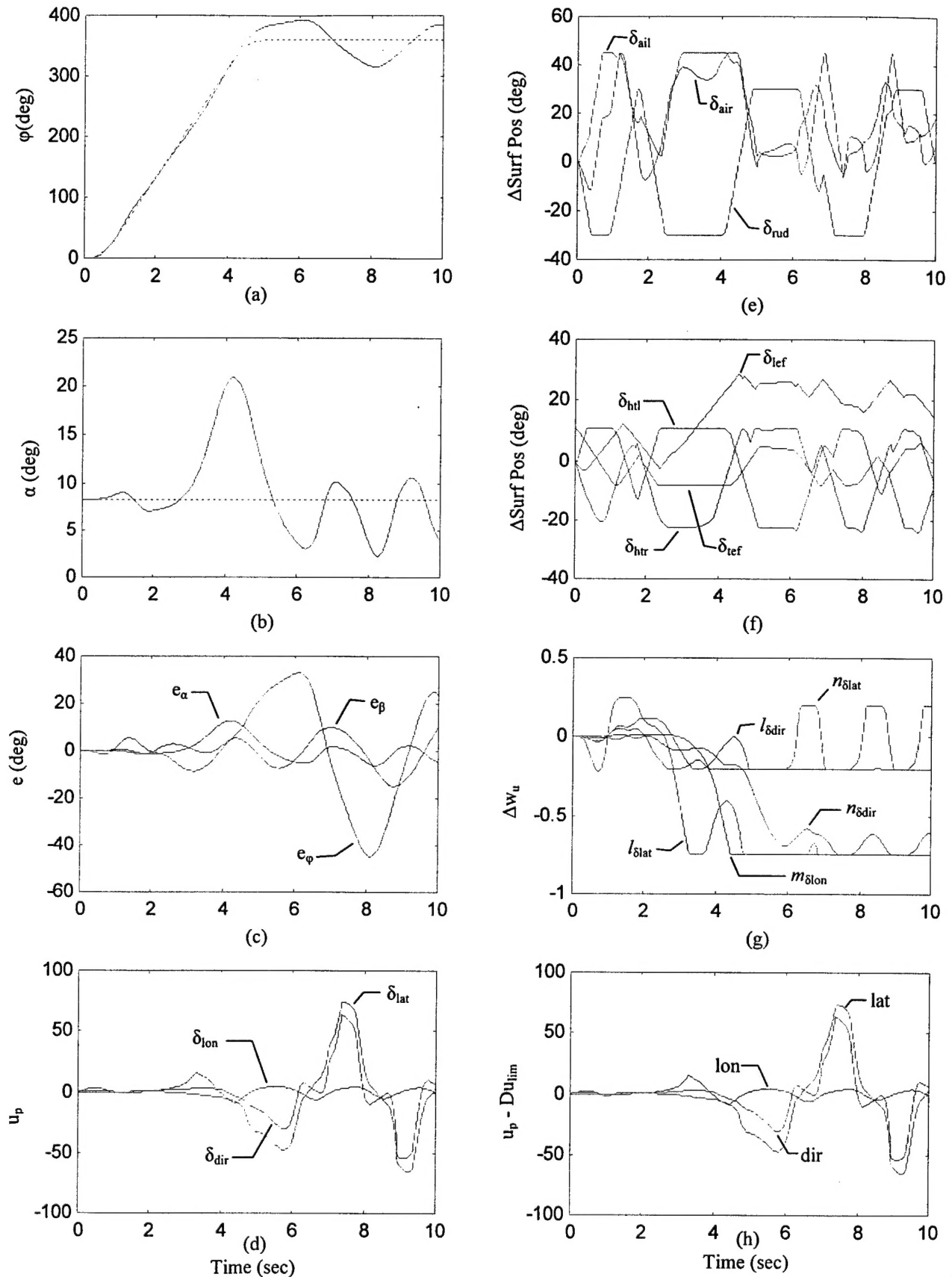**Figure 1: Simulation Results for WPI Allocation**

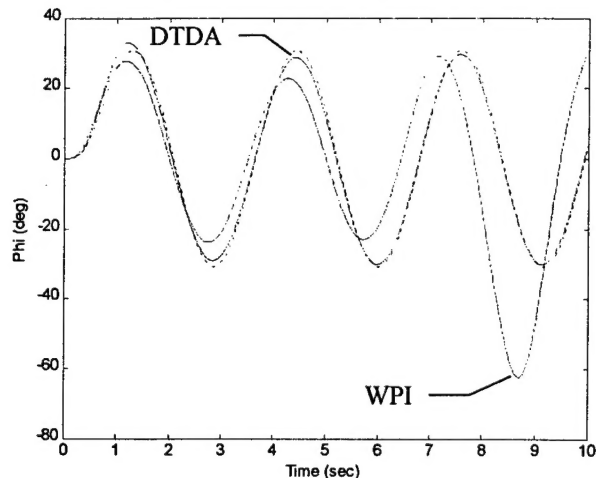**Figure 2: Simulation Results for DTDA Allocation**

Figure 3: Roll Angle Responses for Sinusoidal Input

Conclusions

The effect of the choice of control allocation algorithm used in conjunction with a nonlinear adaptive control law has been studied. The results show that this choice can have dramatic impact on system performance. For the set of maneuvers and initial conditions tested, the Weighted Pseudo-Inverse solution performed well despite producing admissible solutions for only 10% of the Attainable Moment Subset.

The less admirable system performance that results from using Disrcrete Time Direct Allocation with the backstepping adaptive designs is attributed to an important control law/control allocation interaction that results when the effectors become rate or position saturated. Since the situation could have been quite different if the actuator mitigation logic of Reference 8 had been included, these results must not be viewed as a detriment to Direct Allocation. However, the results do serve to illustrate that simply using the best (in terms of ability to meet attainable moment demands) control allocator does not necessarily yield the best system performance. To further complicate matters, the performance results can be sensitive to changes in the maneuver set, the flight conditions, and the initial parameter estimates. Hence, the results presented herein are meant to illustrate the important control law/control allocation interactions rather than to judge the relative merits of the control laws or control allocation techniques on an individual basis.

A better understanding of some of the basic control law/control allocation interactions facilitates an integrated design that endeavors to attain the best performance. Integration of an extended set of control allocation schemes with a broad array of nonlinear control designs is planned for future research.

References

[1]Bugajski, D., Enns, D.,Hendrick, R., Russ, C., A., ., "Flight Control Laws for the HARV," Phase 2 Final Report, March 1992.

[2]Enns, D., Bugajski, D., Hendrick, R., Stein, G., "Dynamic Inversion: An Evolving Methodology for Flight Control Design," *International Journal of Control*, January, 1994.

[3]Buffington, J. M., "Tailless Aircraft Control Allocation," AIAA Paper No. 97-3605, *AIAA Guidance, Navigation, and Control Conference*, New Orleans, LA, Aug. 1997.

[4]Brinker, J., and Wise, K., "Reconfigurable Flight Control for a Tailless Advanced Fighter Aircraft," AIAA Paper No. 98-4107, AIAA Guidance, Navigation, and Control Conference, Boston, MA, 1998.

[5]Enns, D., "Control Allocation Approaches," AIAA Paper No. 98-4109, AIAA Guidance, Navigation, and Control Conference, Boston, MA, 1998.

[6]Kanellakopoulos, I., Kokotovic, P.V., and Morse, A.S., "Systematic Design of Adaptive Controllers for Feedback Linearizable Systems," *IEEE Transactions on Automatic Control*, Vol. 36, 1991, pp. 1241-1253.

[7]Singh, S. N. and Steinberg, M. L., "Adaptive Control of Feedback Linearizable Nonlinear Systems with Application to Flight Control," *Journal of Guidance, Control, and Dynamics*, Vol. 13, No. 6, 1990, pp. 871-877.

[8]Steinberg, M. L., and Page, A. B., "Nonlinear Adaptive Flight Control with a Backstepping Design Approach," AIAA Paper No. 98-4230, AIAA Guidance, Navigation, and Control Conference, Boston, MA, 1998.

[9]Singh, S. N., et al, "Adaptive Control of High-Performance Aircraft with Multiple Control Effectors," Submitted to the AIAA Guidance, Navigation, and Control Conference, Portland, OR, 1999.

[10]Durham, W. C., "Constrained Control Allocation," *Journal of Guidance, Control, and Dynamics*, Vol. 16, No. 4, July-August 1993, pp. 717-725.

[11]Durham, W.C., and Bordignon, K.A., "Multiple Control Effector Rate Limiting," *Journal of Guidance, Control, and Dynamics*, Vol. 19, No. 1, January-February 1996.

[12]Bordignon, K. A., "Constrained Control Allocation for Systems with Redundant Control Effectors," Ph.D. Thesis, Virginia Polytechnic Institute and State University, 1996.

[13]Skelton, R. E., *Dynamic Systems Control*, John Wiley, New York, 1988, pp. 47-48.

[14]Glaze, M., and Durham, W., "Design and Implementation of a Control Allocation Toolbox for MATLAB v. 5.0," AIAA Paper No. 98-4436, AIAA Guidance, Navigation, and Control Conference, Boston, MA, 1998.